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μ-term due to the non-universal supersymmetry breaking and the Higgs mass

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Abstract: We propose a solution to the μ-problem due to the non-universal soft supersymmetry breaking in a class of supersymmetric extensions of the SM with an extra U(1) and the NMSSM. Our scenario resembles to an earlier one which has been proposed as a solution to the μ-problem in the framework of similar models but it has rather different features from that. Theoretically crucial points in the present scenario are the realization of the required non-universal soft supersymmetry breaking parameters and the concern to avoid the vacuum instability due to tadpoles. We discuss them in some detail. In particular, on the non-universal soft supersymmetry breaking we discuss the superstring and the field theory with reduction of couplings possibilities. We also study the weak scale radiative symmetry breaking in the considered models and estimate the upper mass bound of the lightest neutral Higgs scalar. The latter tends to be smaller in the present scenario as compared to the previous one.

Keywords: Supersymmetric Standard Model, Supersymmetry Breaking, Higgs Physics.

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Contents

1. Introduction

Supersymmetry is expected to solve the hierarchy problem in the standard model (SM) [1]. The minimal supersymmetric standard model (MSSM) is the simplest candidate of such models. As is well-known, however, there is a remnant of the hierarchy problem called μ-problem [2]. μ is a supersymmetric mass parameter in the MSSM and it needs to be close to the weak scale for the realization of a correct vacuum. In principle there is no reason why it should be as the weak scale and it is natural for such a supersymmetric mass parameter to be of the order of the unification scale or the Planck scale.

Natural solutions for this problem may be obtained by connecting it with the soft supersymmetry breaking scale. Several scenarios have been proposed in this direction by now [3]–[10]. One of the representative scenarios is to replace a μ-term, $\mu H_1 H_2$ with a Yukawa type coupling $\lambda S H_1 H_2$ by introducing an SM singlet chiral superfield S . This kind of proposal requires also the introduction of some symmetry to prohibit a bare μ-term. The next to the minimal supersymmetric standard model (NMSSM) with a suitable discrete symmetry is an example of such scenarios. In fact, the radiative correction induces a weak scale vacuum expectation value of the scalar component $\tilde{S}$ of S and the μ-scale is realized as $\lambda\langle\tilde{S}\rangle$ [7, 8]. However, in this model the spontaneous breaking of the discrete symmetry creates the cosmologically dangerous domain wall problem. In addition to the bare μ-term problem, the model is also annoyed by the vacuum instability problem if the discrete symmetry is not imposed [11].

We can consider the above proposed symmetry as a gauge symmetry. It has been known that a certain kind of supersymmetric extensions of the SM involving an extra U(1) can solve the μ-problem by replacing the μ-term to a Yukawa coupling among an SM singlet field S and the usual Higgs doublet fields such as $\lambda S H_1 H_2$ [9, 10]. The bare μ-term is forbidden by this extra U(1)-symmetry if S carries non-trivial U(1)-charge. Thus the superpotential of this model has the form

$$W = \lambda S H_1 H_2 + (\text{MSSM Yukawa couplings}) + \cdots . \tag{1.1}$$

Moreover it has been checked that radiative breaking of this U(1)-symmetry could occur by making the soft supersymmetry breaking scalar squared mass of $\tilde{S}$ negative ($m_S^2 < 0$), at least if there were vector like extra colored chiral superfields $(g, \bar{g})$ coupled with S in the form $kSg\bar{g}$ [9, 12, 13]. As in the case of the NMSSM, the coupling is realized as $\lambda\langle\tilde{S}\rangle$. A corresponding soft supersymmetry breaking parameter B is also induced as a usual trilinear soft supersymmetry breaking parameter A_λ corresponding to the $\lambda S H_1 H_2$ term in W.

In this paper we propose a new scenario for solving this problem based on the non-universality of the soft supersymmetry breaking parameters. This scenario is very similar to the one proposed previously in the class of models with an extra U(1) but potentially it can have different features from it. In the general framework in which supergravity describes the low energy effective theory of superstring theories the supersymmetry breaking parameters can appear in a non-universal form [14, 15, 16]. Also a class of renormalizable N=1 supersymmetric gauge theories, in which renormalization group invariant (RGI) relations among couplings have been assumed, have naturally similar features. If soft supersymmetry breaking scalar squared masses are non-universal and some of them are negative, we can consider the possibility that the breaking of the symmetry, which protects the appearance of a bare μ-term, has different features, for example on the Higgs mass bound . It might provide the origin of the μ-scale.

Let us consider the model with the same superpotential as in eq. (1.1). The scalar potential of $\tilde{S}$ is composed only of the extra U(1) D-term and the soft breaking scalar mass term such as

$$V = \frac{g_X^2}{2} Q_S^2 |\tilde{S}|^4 + \tilde{m}_S^2 |\tilde{S}|^2 , \tag{1.2}$$

where g_X is the extra U(1) gauge coupling and Q_S is the corresponding charge of S. If $\tilde{m}_S^2 < 0$ is radiatively realized at some scale satisfying the positiveness of other masses, the scalar component of S obtains the vacuum expectation value $|\langle\tilde{S}\rangle|^2 = -\tilde{m}_S^2/(g_X Q_X)^2$. In this case the vacuum expectation value $\langle\tilde{S}\rangle$ is always expected to be around the supersymmetry breaking scale. Even if the negative $\tilde{m}_S^2$ is introduced as a result of non-universal supersymmetry breaking at the unification scale M_X, this estimation will be kept. However in the latter case the μ-term may be considered to be induced at the unification scale as a direct result of the non-universality of the soft supersymmetry breaking. In order to find out the features arising in such a case at the weak scale we need a study using the renormalization group equations. The components of the extra U(1)-vector superfields do not decouple until the weak scale. So the phenomenology at the weak scale may be very similar to the usual low energy breaking scenario except that there is, in principle, no need to introduce large coupling of extra colored fields g and $\bar{g}$ to S and therefore the result of the running of the renormalization group equations (RGEs) is different from the usual case [9, 10].

Crucial points for the success of the present scenario will be a consistent realization of the non-universal soft scalar masses required and a solution of the problem of the vacuum instability due to the non-renormalizable interaction terms of S in the frameworks that they appear [11]. It is also interesting to examine what kind of phenomenological differences appear between the $\tilde{m}_S^2 > 0$ case and the $\tilde{m}_S^2 < 0$ case. The same scenario is applicable to the NMSSM.

The organization of this paper is as follows. In section 2 we will briefly review the general structure of the soft supersymmetry breaking parameters in the effective supergravity of superstring. In section 3 we will present some examples which are expected to realize the above scenario and discuss some important points concerning the relevant models. In section 4 we present the field theoretical framework that accommodates the non-universal soft supersymmetry breaking scalar mass squared terms. In section 5 the mass bound of the lightest neutral Higgs for the model with an extra U(1) and the NMSSM is discussed as a representative example of the features that discriminate this class of models from others. Section 6 is devoted to the summary.

2. Non-universal supersymmetry breaking in the effective supergravity of superstring

Non-universality of the soft scalar mass is an essential point in our scenario. It will be useful to start with a brief review of the gravity mediated supersymmetry breaking [14, 15, 16]. Low energy effective supergravity theory of superstring is characterized in terms of the Kähler potential K, the superpotential W and the gauge kinetic function f_a. Each of these is a function of ordinary massless chiral matter superfields Ψ^I and gauge singlet fields Φ^i called moduli,[1] whose potential is perturbatively flat as far as the supersymmetry is unbroken. We assume here that the non-perturbative phenomena such as gaugino condensation occur in a hidden sector. After integrating out the fields relevant to these phenomena, the Kähler potential and the superpotential are expanded in the low energy observable matter

[1]We are using the terminology "moduli" in the generalized meaning. A dilaton $\mathcal{S}(\equiv \Phi^0)$ is included in Φ^i other than the usual moduli $M^i(\equiv \Phi^i\ (1 \leq i \leq N))$.

fields Ψ^I as,[2]

$$K = \kappa^{-2}\hat{K}(\Phi,\bar{\Phi}) + Z(\Phi,\bar{\Phi})_{I\bar{J}}\Psi^I\bar{\Psi}^{\bar{J}} + \cdots\,, \tag{2.1}$$

$$W = \hat{W}(\Phi) + \frac{1}{3}\tilde{h}(\Phi)_{IJK}\Psi^I\Psi^J\Psi^K + \cdots, \tag{2.2}$$

where $\kappa^2 = 8\pi/M_{\rm pl}^2$. The ellipses stand for the terms of higher orders in Ψ^I. In eq. (2.2), $\hat{W}(\Phi)$ is considered to be induced by the non-perturbative effects in the hidden sector. Using these functions the scalar potential V can be written as [17],

$$V = \kappa^{-2}e^G\left[G_\alpha(G^{-1})^{\alpha\bar{\beta}}G_{\bar{\beta}} - 3\kappa^{-2}\right] + ({\rm D-term})\,, \tag{2.3}$$

where $G = K + \kappa^{-2}\log\kappa^6|W|^2$ and the indices α and β denote Ψ^I as well as Φ^i. The gravitino mass $m_{3/2}$ which characterizes the scale of supersymmetry breaking is expressed as $m_{3/2} = \kappa^2 e^{\hat{K}/2}|\hat{W}|$. In order to get the soft supersymmetry breaking terms in the low energy effective theory from eq. (2.3), we take the flat limit $M_{\rm pl}\to\infty$ preserving $m_{3/2}$ fixed. Through this procedure we obtain the effective renormalizable superpotential $W^{\rm eff}$ and the soft supersymmetry breaking terms $\mathcal{L}_{\rm soft}$. However, in the real world $M_{\rm pl}\neq\infty$ and then we have a tower of non-renormalizable terms suppressed by powers of $M_{\rm pl}$. These non-renormalizable terms could induce a vacuum instability as discussed in ref. [11]. We will come back to this point later.

In the effective superpotential $W^{\rm eff}$ Yukawa couplings are rescaled as $h_{IJK} = e^{\hat{K}/2}\tilde{h}_{IJK}$. The soft breaking terms $\mathcal{L}_{\rm soft}$ corresponding to $W_{\rm eff}$ is defined by

$$\mathcal{L}_{\rm soft} = -\tilde{m}^2_{I\bar{J}}\psi^I\bar{\psi}^{\bar{J}} - \left(\frac{1}{3}A_{IJK}\psi^I\psi^J\psi^K + {\rm h.c.}\right), \tag{2.4}$$

where ψ^I represents the scalar component of Ψ^I. Bilinear terms of Ψ_I are assumed to be forbidden by some symmetry. Each soft breaking parameter is expressed by using K and W as follows [15],[3]

$$\tilde{m}^2_{I\bar{J}} = m^2_{3/2}Z_{I\bar{J}} - F^i\bar{F}^{\bar{j}}\left[\partial_i\partial_{\bar{j}}Z_{I\bar{J}} - \left(\partial_{\bar{j}}Z_{N\bar{J}}\right)Z^{N\bar{L}}\left(\partial_i Z_{I\bar{L}}\right)\right] + \kappa^2 V_0 Z_{I\bar{J}}\,, \tag{2.5}$$

$$A_{IJK} = F^i\left[\left(\partial_i + \frac{1}{2}\hat{K}_i\right)h_{IJK} - Z^{\bar{M}L}\partial_i Z_{\bar{M}(I}h_{JK)L}\right], \tag{2.6}$$

where F^i is an F-term of Φ^i and ∂_i denotes $\partial/\partial\Phi^i$. V_0 is the cosmological constant expressed as $V_0 = \kappa^{-2}(F^i\bar{F}^{\bar{j}}\partial_i\partial_{\bar{j}}\hat{K} - 3m^2_{3/2})$. From these expressions we find that the soft breaking parameters are generally non-universal and their structure is dependent

[2]Here we do not assume the existence of the effective μ-term in W [4] and also the Giudice-Masiero term in K which can become the origin of μ-term [5].

[3]These soft breaking parameters are not canonically normalized because a kinetic term of ψ^I is expressed as $Z_{I\bar{J}}\partial^\mu\psi^I\partial_\mu\bar{\psi}^{\bar{J}}$.

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on the form of Kähler potential, especially, the functional form of $Z_{I\bar{J}}$. The canonically normalized gaugino mass is known to be written by using the gauge kinetic function $f_a(\Phi)$ as [17]

$$M_a = \frac{1}{2}(\mathrm{Re} f_a)^{-1} F^i \partial_i f_a \,. \tag{2.7}$$

Now we apply the above formalism to our model. The chiral superfields Ψ^I represent quarks and leptons $Q^\alpha, \bar{U}^\alpha, \bar{D}^\alpha, L^\alpha$ and $\bar{E}^\alpha$ etc. where α is a generation index. Then, the effective superpotential $W^{\rm eff}$ and soft supersymmetry breaking terms $\mathcal{L}_{\rm soft}$ in our model can be written as

$$W^{\rm eff} = h^U_{\alpha\beta H_2} \bar{U}^\alpha H_2 Q^\beta + h^D_{\alpha\beta H_1} \bar{D}^\alpha H_1 Q^\beta + h^E_{\alpha\beta H_1} \bar{E}^\alpha H_1 L^\beta + \lambda S H_1 H_2 + k S g \bar{g}, \tag{2.8}$$

$$\mathcal{L}_{\rm soft} = -\sum_{I,J} z^{I\dagger} \tilde{m}^2_{\bar{I}J} z^J - \Big(A^U_{\alpha\beta H_2} \bar{U}^\alpha H_2 Q^\beta + A^D_{\alpha\beta H_1} \bar{D}^\alpha H_1 D^\beta + A^E_{\alpha\beta H_1} \bar{E}^\alpha H_1 L^\beta + + A_\lambda S H_1 H_2 + A_k S g \bar{g} + \text{h.c.} \Big), \tag{2.9}$$

where we do not consider the gaugino masses which are irrelevant in our present discussion. The bare μ-term in $W^{\rm eff}$ is forbidden by the extra U(1)-symmetry. The first term of eq. (2.9) represents the mass terms of all scalar components of chiral superfields ($z^I = Q^\alpha, U^\alpha, D^\alpha, L^\alpha, E^\alpha, H_1, H_2, S, g, \bar{g}$) in our model. The first place to check the validity of our scenario is to study whether we can construct the model in which the soft scalar masses are consistently realized in the required way such as

$$\tilde{m}^2_S < 0\,, \qquad \tilde{m}^2_{H_1} > 0\,, \qquad \tilde{m}^2_{H_2} > 0\,, \qquad \tilde{m}^2_{\hat{I}} > 0\,, \tag{2.10}$$

where $\hat{I} = Q, \bar{U}, \bar{D}, L, \bar{E}, g, \bar{g}$. As seen from the general expressions of soft breaking parameters in eqs. (2.5) and (2.6), their structure is determined by the moduli dependence of $Z_{I\bar{J}}$ and W.[4]

In order to apply the above general results to construct a model satisfying eq. (2.10) and proceed further investigation of the validity of our scenario, it is necessary to make the model more definite by introducing some assumptions [18]. We consider a concrete model within the framework which satisfies the following conditions. At first we assume the simplest target space duality SL$(2,\mathbb{Z})$ invariance [14]

$$M^i \to \frac{a_i M^i - i b_i}{i c_i M^i + d_i} \qquad (a_i d_i - b_i c_i = 1,\ a_i, b_i, c_i, d_i \in \mathbb{Z}), \tag{2.11}$$

for each usual modulus M^i. Under the target space duality transformations (2.11) the chiral superfields Ψ^I are assumed to transform as

$$\Psi^I \to \left(i c_i M^i + d_i\right)^{n^i_I} \Psi^I\,, \tag{2.12}$$

[4]Only known exception is the dilaton dominated supersymmetry breaking. Our scenario cannot be applicable to this case.

where n_I^i is called the modular weight and takes a suitable rational value. This requirement also results in the invariance under the following Kähler transformation,

$$K \to K + f(M^i) + \bar{f}(\bar{M}^{\bar{i}}), \qquad W \to e^{-f(M^i)} W \,. \tag{2.13}$$

Additionally Kähler metric and then the kinetic terms of chiral superfields are assumed to be flavor diagonal $Z_{I\bar{J}} = Z_I \delta_{IJ}$. This is satisfied almost in all known superstring models as suggested in [16].

In order to parametrize the direction of supersymmetry breaking in the moduli space, we introduce the parameters Θ_i which correspond to the generalized Goldstino angles in the moduli space [16, 19]. They are defined as

$$F^i \sqrt{\hat{K}_{i\bar{j}}} = \sqrt{3} C m_{3/2} \Theta_i, \qquad \sum_{i=0}^{N} \Theta_i^2 = 1 \,, \tag{2.14}$$

where we take the $\kappa = 1$ unit. N is the number of usual moduli M^i in the models. A constant C satisfies $V_0 = 3\kappa^{-2}(|C|^2 - 1) m_{3/2}^2$. In the followings we assume $C = 1$ and then $V_0 = 0$. The introduction of these parameters makes it possible to discuss the soft breaking parameters without referring to the origin of supersymmetry breaking.

In the presently known perturbative superstring models the $\hat{K}$ can be generally written as

$$\hat{K} = -\sum_{i=0}^{N} \ln \left(\Phi^i + \bar{\Phi}^{\bar{i}} \right) . \tag{2.15}$$

If we apply the above assumptions to $Z_{I\bar{J}}$ in the Kähler potential K, we can constrain the functional form of Z_I as

$$Z_I = \prod_{i=1}^{N} (M^i + \bar{M}^{\bar{i}})^{n_I^i} . \tag{2.16}$$

Using these facts in eqs. (2.5) and (2.6) and normalizing them canonically, we can write down the soft breaking parameters as[5]

$$\tilde{m}_I^2 = m_{3/2}^2 \left(1 + 3 \sum_{i=1}^{N} \Theta_i^2 n_I^i \right) , \tag{2.17}$$

$$\frac{A_{\hat{I}\hat{J}H_\alpha}}{h_{\hat{I}\hat{J}H_\alpha}} = -\sqrt{3} m_{3/2} \sum_{i=0}^{N} \Theta_i \left[n_{\hat{I}}^i + n_{\hat{J}}^i + n_{H_\alpha}^i + 1 - (M_i + M_i^*) \frac{\partial_i \tilde{h}_{\hat{I}\hat{J}H_\alpha}}{\tilde{h}_{\hat{I}\hat{J}H_\alpha}} \right] , \tag{2.18}$$

$$\frac{A_\lambda}{\lambda} = -\sqrt{3} m_{3/2} \sum_{i=0}^{N} \Theta_i \left[n_S^i + n_{H_1}^i + n_{H_2}^i + 1 - (M_i + M_i^*) \frac{\partial_i \tilde{\lambda}}{\tilde{\lambda}} \right] , \tag{2.19}$$

[5]It should be noted that these are the tree level results. However, the introduction of string one-loop effects [20] will not change the qualitative features discussed here.

where H_α stands for H_1 and H_2. An index I represents all chiral superfields and $\hat{I}$ and $\hat{J}$ represent quarks and leptons $Q_\alpha, \bar{U}_\alpha, \bar{D}_\alpha, L_\alpha$ and $\bar{E}_\alpha(\alpha = 1 \sim 3)$. Here we do not display the expression of A_k which is similar to the one of other A-parameters. In these formulae $n_I^0 = 0$ since we do not consider the transformations such as eqs. (2.11) and (2.12) for a dilaton. There is the gaugino mass as a remaining soft supersymmetry breaking parameter. Recalling that the gauge kinetic function is known that can be written as $f_a = k_a \mathcal{S}$ and using eq. (2.7), we obtain

$$M_a = \sqrt{3}\Theta_0 m_{3/2} \,, \tag{2.20}$$

where we neglect the threshold correction to f_a.

Taking account of the functional form of $\hat{K}$ in eq. (2.15), our assumption requires $f(M^i) = -\ln(ic_i M^i + d_i)$ in eq. (2.13). As a result, eq. (2.13) shows that the transformation property of superpotential W is: $W \to \prod_{i=1}^N (ic_i M^i + d_i)^{-1} W$. Thus its coefficient function $\tilde{h}_{IJK}$ transforms as the modular forms under the duality transformation of the moduli fields M^i,

$$\tilde{h}_{IJK} \to \prod_{i=1}^N \left(ic_i M^i + d_i\right)^{-(n_I^i + n_J^i + n_K^i + 1)} \tilde{h}_{IJK} \,. \tag{2.21}$$

This shows that $\tilde{h}_{IJK}$ is a modular function of M_i with modular weight the $-(n_I^i + n_J^i + n_K^i + 1)$. If coefficient functions $\tilde{h}_{IJK}$ of the superpotential are independent of the moduli fields whose F-terms contribute to the supersymmetry breaking: $\partial_i \tilde{h}_{IJK} = 0$ for $\langle F^i \rangle \neq 0$, then the superpotential W, except for a $\hat{W}$ part, can depend only on moduli which do not contribute the supersymmetry breaking. Then the last terms in eqs. (2.18) and (2.19) have no contribution.[6] In such a case the relation

$$n_I^i + n_J^i + n_K^i + 1 = 0 \,, \tag{2.22}$$

should be fulfilled for each $i(\neq 0)$. When this condition is satisfied, we can easily check that one of the sum rules [18, 22] is satisfied as

$$\tilde{m}_I^2 + \tilde{m}_J^2 + \tilde{m}_K^2 = \left(\frac{A_{IJK}}{h_{IJK}}\right)^2 = M_a^2 \,. \tag{2.23}$$

However, even if $\tilde{h}_{IJK}$ is a nontrivial modular function of M_i whose auxiliary component has non-zero VEV $\langle F^i \rangle \neq 0$, such a situation as $\partial_i \tilde{h}_{IJK} \ll \tilde{h}_{IJK}$ can happen. In such a case the last term in eqs. (2.18) and (2.19) can be neglected again, although the sum rule (2.23) may not be satisfied unless we consider the suitable supersymmetry breaking direction Θ_i. Although we assume that the last term in eqs. (2.18) and (2.19) is negligible in the following discussion, we will take account of both possibilities on this sum rule.

[6] In this special situation such Yukawa couplings $\tilde{h}_{IJK}$ can be dynamical variables at the low energy region as discussed in ref. [21]. It may be a phenomenologically interesting case.

3. Negative scalar mass squared

In the previous section we saw that the structure of soft supersymmetry breaking parameters are determined by the supersymmetry breaking direction Θ_i and modular weights of the matter fields. If we use these formulas, we can write down the required condition (2.10) for them to realize our scenario consistently. It is rewritten as

$$\sum_{i=1}^{N}\Theta_i^2 n_S^i < -\frac{1}{3}\,, \qquad \sum_{i=1}^{N}\Theta_i^2 n_{H_\alpha}^i > -\frac{1}{3}\,, \qquad \sum_{i=1}^{N}\Theta_i^2 n_{\hat{I}}^i > -\frac{1}{3}\,, \tag{3.1}$$

where $\hat{I}$ stands for the fields including the MSSM contents other than H_α and S. It is instructive to give some concrete examples which satisfy this condition in the framework of perturbative superstring. As representative examples, we consider two models which have an overall modulus and multi moduli, respectively.

The first example is a model with an overall modulus ($N = 1$). In this case the soft scalar mass formula reduces to $\tilde{m}_I^2 = m_{3/2}^2(1 + \Theta_1^2 n_I)$. If all the matter fields have $n_I = -1$, we have universal soft supersymmetry breaking parameters such as

$$\tilde{m}_I^2 = m_{3/2}^2\Theta_0^2\,, \qquad \frac{A_{IJK}}{h_{IJK}} = -\sqrt{3}m_{3/2}\Theta_0\,, \qquad M_a = \sqrt{3}m_{3/2}\Theta_0\,. \tag{3.2}$$

This structure is the same as in the celebrated dilaton dominated case. Now we consider the necessary change to realize our scenario. The condition (3.1) becomes $\Theta_1^2 n_S < -1$ for S and $\Theta_1^2 n_{\hat{I}} > -1$ for other fields including the MSSM contents. In order to satisfy them, we must modify the modular weight of S at least. As the simplest modification we change the modular weight of S into $n_S = -2$. The universality among the soft scalar masses of the ordinary MSSM contents is kept by this change. The soft scalar masses and A-parameters are shifted into

$$\begin{aligned} \tilde{m}_S^2 &= m_{3/2}^2(\Theta_0^2 - \Theta_1^2), \qquad \tilde{m}_{\hat{I}}^2 = m_{3/2}^2\Theta_0^2\,, \\ \frac{A_\lambda}{\lambda} &= -\sqrt{3}m_{3/2}\left(\Theta_0 - \frac{\Theta_1}{3}\right), \qquad \frac{A_{\hat{I}\hat{J}\hat{K}}}{h_{\hat{I}\hat{J}\hat{K}}} = -\sqrt{3}m_{3/2}\Theta_0\,, \end{aligned} \tag{3.3}$$

where the suffix $\hat{I}$ includes the ordinary Higgs doublet fields. The A-parameter corresponding to $kSg\bar{g}$ term has the same expression as A_λ. In this situation our scenario will be realized as far as $1/2 < \Theta_1^2 < 1$. If we tend Θ_1^2 to one, $\tilde{m}_S^2$ becomes much more negative. In this example the condition (2.22) is satisfied in the ordinary MSSM Yukawa couplings. On the other hand, since the λ-term does not satisfy it, there appears a Θ_1 dependence as can be easily seen from eq. (2.19). In general, for the ordinary Yukawa couplings in the superpotential it is well-known that the following condition on the soft supersymmetry breaking parameters is required to prohibit the undesired symmetry breaking [8]:[7]

$$\tilde{m}_I^2 + \tilde{m}_J^2 + \tilde{m}_K^2 > \frac{1}{3}\left(\frac{A_{IJK}}{h_{IJK}}\right)^2 . \tag{3.4}$$

We should check the validity of this condition for the ordinary MSSM Yukawa couplings to quarantee the consistency of our scenario. However, it is trivially satisfied at least at the unification scale since the sum rule (2.23) holds in this example.

As a next example, we adopt the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with three independent moduli (T_1, T_2, T_3) [14]. In this orbifold we can consider a model composed of the fields in the untwisted sector with modular weights $(-1,0,0)$, $(0,-1,0)$, $(0,0,-1)$ and the fields in the twisted sector with modular weight $(-1/2,-1/2,0)$. Here we assume that the modular weights of H_1, H_2 and S are $(-1,0,0)$, $(0,-1,0)$ and $(0,0,-1)$ respectively and other matter contents in the MSSM and $(g,\bar{g})$ have $(-1/2,-1/2,0)$. This assignment guarantees the universality of the soft scalar masses among the MSSM contents except for the doublet Higgs fields. Under this assumption we can write down the soft supersymmetry breaking parameters $\tilde{m}_I^2$ and A_{IJK} as follows,

$$\begin{aligned}
&\tilde{m}_{\hat{I}}^2 = m_{3/2}^2\left(1-\frac{3}{2}(\Theta_1^2+\Theta_2^2)\right), \quad \tilde{m}_{H_\alpha}^2 = m_{3/2}^2\left(1-3\Theta_\alpha^2\right), \quad \tilde{m}_S^2 = m_{3/2}^2\left(1-3\Theta_3^2\right), \\
&\frac{A_{\hat{I}\hat{J}H_\alpha}}{h_{\hat{I}\hat{J}H_\alpha}} = -\sqrt{3}m_{3/2}\left(\Theta_0 - \Theta_\alpha + \Theta_3\right), \qquad \frac{A_\lambda}{\lambda} = \frac{A_k}{k} = -\sqrt{3}m_{3/2}\Theta_0 ,
\end{aligned} \tag{3.5}$$

where the index α takes 1 and 2. A_λ corresponds to the B-parameter in the MSSM. At this stage the structure of soft supersymmetry breaking parameters is completely determined by the supersymmetry breaking direction Θ_i. Here we might note that eq. (2.22) is satisfied only in the λ-term in this modular weight assignment. The ordinary MSSM Yukawa couplings do not satisfy eq. (2.22) and then it is not trivial whether the condition (3.4) can be satisfied or not. The breaking direction Θ_i should be constrained by requiring to satisfy it. It is useful to note that the gaugino mass and then $\tilde{m}_{H_1}^2 + \tilde{m}_{H_2}^2 + \tilde{m}_S^2$ becomes smaller by making Θ_0 smaller.

In this example the supersymmetry breaking pattern similar to the dilaton dominance is realized by $\Theta = (1/\sqrt{3}, -\sqrt{2}/3, -\sqrt{2}/3, -\sqrt{2}/3)$ and each parameter is represented by

$$\begin{aligned}
&\tilde{m}_{\hat{I}}^2 = \tilde{m}_{H_\alpha}^2 = \tilde{m}_S^2 = \frac{1}{3}m_{3/2}^2 , \qquad M_a = m_{3/2} , \\
&\frac{A_{\hat{I}\hat{J}H_\alpha}}{h_{\hat{I}\hat{J}H_\alpha}} = \frac{A_\lambda}{\lambda} = \frac{A_k}{k} = -m_{3/2} .
\end{aligned} \tag{3.6}$$

This satisfies the sum rule (2.23). If we change Θ_i from this case, we can introduce the non-universality among soft scalar masses. It is not difficult to make $\tilde{m}_{H_1}^2$ different

[7] Of course this condition should be satisfied at the weak scale.

from $\tilde{m}^2_{H_2}$ keeping their positivity. In fact, if we take $\Theta = (1/\sqrt{6}, -\sqrt{2}/3, -\sqrt{11}/6, -\sqrt{11}/6)$, we obtain a non-universal example such as

$$\begin{aligned} &\tilde{m}^2_{\hat{I}} = \frac{5}{24} m^2_{3/2}\,, \quad \tilde{m}^2_{H_1} = \frac{1}{3} m^2_{3/2}\,, \quad \tilde{m}^2_{H_2} = \frac{1}{12} m^2_{3/2}\,, \quad \tilde{m}^2_S = \frac{1}{12} m^2_{3/2}\,, \\ &M_a = \frac{1}{\sqrt{2}} m_{3/2}\,, \quad \frac{A_{\hat{I}\hat{J}H_\alpha}}{h_{\hat{I}\hat{J}H_\alpha}} = \frac{A_\lambda}{\lambda} = \frac{A_k}{k} = -\frac{1}{\sqrt{2}} m_{3/2}\,. \end{aligned} \tag{3.7}$$

Now the first condition of eq. (3.1) is rewritten as $\Theta_3^2 > 1/3$. As an simple way to satisfy this condition, we shift the above Θ into $\Theta = (1/3, -\sqrt{2}/3, -\sqrt{2}/3, -2/3)$. This choice keeps the soft scalar masses the same as in eq. (3.6) except for $\tilde{m}^2_S$ and they are expressed as

$$\begin{aligned} &\tilde{m}^2_{\hat{I}} = \tilde{m}^2_{H_\alpha} = -\tilde{m}^2_S = \frac{1}{3} m^2_{3/2}\,, \qquad M_a = \frac{1}{\sqrt{3}} m_{3/2}\,, \\ &\frac{A_{\hat{I}\hat{J}H_\alpha}}{h_{\hat{I}\hat{J}H_\alpha}} = -\frac{\sqrt{2}-1}{\sqrt{3}} m_{3/2}\,, \qquad \frac{A_\lambda}{\lambda} = \frac{A_k}{k} = -\frac{1}{\sqrt{3}} m_{3/2}\,. \end{aligned} \tag{3.8}$$

Note that with the above chosen Θ's which led to eqs. (3.6) and (3.7), the sum rule (2.23) is partially satisfied according to the discussion in section 2. In fact, in eq. (3.7) the sum rule is broken in the down sector and in eq. (3.8) it is broken in the λ-term. We can easily introduce more non-universality among these parameters by changing Θ from this.

Here we presented simple examples. However, as it is clearly seen from these examples, we can find that it is not difficult to prepare the set of parameters in a way that our scenario could be potentially realized. Athough all soft parameters are generally of the same order in the present scheme, even within these examples we find that rather extensive and phenomenologically allowable non-universal soft supersymmetry breaking structure can be realized. It is useful to note that their hierarchical structure can be also realized if we take into account the one-loop effect [19]. Moreover, if we extend our consideration to a set of modular weights which have not been known to be realized in the concrete superstring models, we can construct many models with different patterns of the soft breaking parameters. We can also expect that this scenario might be straightfowardly extended to the soft supersymmetry breaking parameters in the M-theory framework.

4. Reduction of couplings and soft scalar masses sum rule

The above proposed scenario can also be realised in a simpler field theoretical framework of supersymmetric GUTs in which *reduction of couplings* has been achieved. The method of reducing the couplings consists of hunting for renormalization group invariant (RGI) relations. This programme, called Gauge-Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and

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Yukawa couplings, had already noticeable successes by predicting correctly, among others, the top quark mass in the finite and in the minimal N=1 supersymmetric SU(5) GUTs [23, 24]. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of reduction of couplings [25]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [26, 27].

Let us outline briefly the idea of reduction of couplings. Any RGI relation among couplings (which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \cdots, g_A) \;=\;$ const., which has to satisfy the partial differential equation (PDE)

$$\mu\,\frac{d\Phi}{d\mu} = \vec{\nabla}\cdot\vec{\beta} = \sum_{a=1}^{A}\beta_a\,\frac{\partial\Phi}{\partial g_a} = 0\,, \tag{4.1}$$

where β_a is the β-function of g_a. This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [25],

$$\beta_g\,\frac{dg_a}{dg} = \beta_a\ ,\ \ a = 1, \ldots, A\ , \tag{4.2}$$

where g and β_g are the primary coupling and its β-function, and the counting on a does not include g. Since maximally $(A-1)$ independent RGI "constraints" in the A-dimensional space of couplings can be imposed by the Φ_a's, one could in principle express all the couplings in terms of a single coupling g. The strongest requirement is to demand power series solutions to the REs,

$$g_a = \sum_{n=0}\rho_a^{(n)}\,g^{2n+1}\,, \tag{4.3}$$

which formally preserve perturbative renormalizability. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [25].

The method of reducing the dimensionless couplings has been extended [28] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N=1$ supersymmetric theories. In addition it was found [29] that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule. Let us briefly describe here how the use of the available two-loop RG functions and the requirement of finiteness of the SSB parameters up to this order leads to the soft scalar-mass sum rule [30].

Consider the superpotential given by[8]

$$W = \frac{1}{2}\,m_{ij}\,\Phi_i\,\Phi_j + \frac{1}{6}\,C_{ijk}\,\Phi_i\,\Phi_j\,\Phi_k\,, \tag{4.4}$$

[8]In this section we change the notation for the Yukawa couplings and the soft supersymmetry breaking parameters from the previous section as follows: $h_{IJK} \to C_{ijk},\ \ A_{IJK} \to h_{ijk},\ \ \tilde{m}^2_{I\bar{J}} \to (m^2)^j_i$.

where m_{ij} and C_{ijk} are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. The Lagrangian for SSB terms along this superpotential is

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\, (m^2)^{j}_{i}\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}\,, \tag{4.5}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Considering first finite theories, we assume that the gauge group is a simple group and the one-loop β-function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n}\,. \tag{4.6}$$

According to the finiteness theorem [26], the theory is then finite to all-orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j\,(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by

$$h^{ijk} = -MC^{ijk} + \ldots = -M\rho^{ijk}_{(0)}\, g + O(g^5)\,. \tag{4.7}$$

Now, to obtain the two-loop sum rule for soft scalar masses which guarantees together with condition (4.7) the finiteness of the theory in its SSB sector, we assume that the lowest order coefficients $\rho^{ijk}_{(0)}$ and also $(m^2)^i_j$ satisfy the diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \text{ for all } p \text{ and } q \quad \text{and} \quad (m^2)^i_j = m^2_j \delta^i_j\,, \tag{4.8}$$

respectively. Then we find the following soft scalar-mass sum rule

$$\frac{(\ m^2_i + m^2_j + m^2_k\ )}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\, \Delta^{(1)} + O(g^4) \tag{4.9}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction

$$\Delta^{(1)} = -2\sum_l \left[\left(\frac{m^2_l}{MM^\dagger}\right) - \left(\frac{1}{3}\right)\right] T(R_l)\,, \tag{4.10}$$

which vanishes for the universal choice in accordance with previous findings [31].

If we know higher-loop β-functions explicitly, we can follow the same procedure and find higher-loop RGI relations among SSB terms. However, the β-functions of the soft scalar masses are explicitly known only up to two loops. In order to obtain higher-loop results, we need something else instead of knowledge of explicit β-functions, e.g. some relations among β-functions.

The recent progress made using the spurion technique [32, 33] leads to the following all-loop relations among SSB β-functions [34]–[39].

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right), \tag{4.11}$$

$$\beta_h^{ijk} = \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} - 2\gamma^i_{1l} C^{ljk} - 2\gamma^j_{1l} C^{ilk} - 2\gamma^k_{1l} C^{ijl}, \tag{4.12}$$

$$(\beta_{m^2})^i{}_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j, \tag{4.13}$$

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right), \tag{4.14}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}}, \tag{4.15}$$

where $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $C_{lmn} = (C^{lmn})^*$, and

$$\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl}\,. \tag{4.16}$$

The X in eq. (4.12) in the lowest order is

$$X^{(2)} = -\frac{Sg^3}{8\pi^2}, \quad S\delta_{AB} = (m^2)^k_l (R_A R_B)^l_k - |M|^2 C(G)\delta_{AB}\,. \tag{4.17}$$

Then assuming more generally than the finite case, described above, (a) the existence of a RGI surface on which $C = C(g)$ or equivalently that

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{4.18}$$

holds, and (b) the existence of RGI surface on which equation

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g}\,, \tag{4.19}$$

holds too and using the all-loop gauge β-function of Novikov et al. [40] given by

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C(G)}{1 - g^2 C(G)/8\pi^2}\right], \tag{4.20}$$

it was found the all-loop RGI sum rule [42],

$$\begin{aligned} m_i^2 + m_j^2 + m_k^2 = |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2} \right\} + \\ + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g}. \end{aligned} \tag{4.21}$$

The lowest order relations take very simple form and we may summarize them as[9]

$$m_i^2 + m_j^2 + m_k^2 = M^2 \,, \tag{4.22}$$

$$h^{ijk} = -MC^{ijk} \,. \tag{4.23}$$

It is interesting enough to note that these formulae have the same form as eq. (2.23). These formulae can be used directly in applications using GUTs in which has been assumed reduction of coupling beyond the unification point [30].

5. Phenomenological features

In the previous sections it was shown that we could construct the models which potentially induced the μ-scale at the high energy region such as the unification scale. In this section we study the lightest neutral Higgs scalar mass bound as one of their phenomenological features. Before proceeding this study we start our discussion with the vacuum instability problem due to the non-renormalizable terms in the Langangian. It has been known that there appears a vacuum instability in the models with a singlet scalar chiral superfield if we take account of the gravity effects [11]. In such models it has been shown, taking the cutoff scale Λ to be of order $M_{\rm pl}$, that the dangerous divergent diagram is propotional to $M_{\rm pl}^{2-E_c-P_c}$ where E_c stands for the number of external lines and P_c represents the number of chiral propagators. If there is no symmetry to prohibit the tadpole, then the tadpole contribution with $E_c = 1$ violates the weak scale stability. Although in our scenario the chiral superfield S is an SM singlet in the case of the model with an extra U(1), it has a non-trivial charge under the U(1). The divergence is at most logarithmic in the case of $E_c = 2$ and the vacuum instability seems to be escapable due to the extra U(1)-symmetry. This situation does not change even if this symmetry breaks down, as in our model, at $M_{\rm pl}$. In the NMSSM with a discrete symmetry the same situation can be expected.

The electroweak radiative symmetry breaking can also be one of the crucial check points for the validity of the proposed scenario. Although the μ-scale is induced at the unification scale in the present scheme, the models cannot be reduced to the MSSM at the unification scale. Since the vacuum expectation value of $\tilde{S}$ is comparable to the weak scale, this chiral superfield cannot be integrated out to replace the Yukawa coupling $\lambda S H_1 H_2$ by an effective μ-term at such a high energy scale. This singlet chiral superfield S does not decouple until near the weak scale. Thus we should treat our models in the same way as the usual ones with $\tilde{m}_S^2 > 0$. However, it is expected that there is a large difference in the allowed parameter space between the models with a different sign of $\tilde{m}_S^2$ since the couplings behavior required to realize

[9]Here we neglect the new CP violating phases in the soft breaking parameters.

the correct vacuum radiatively is not the same. This difference can be reflected in the mass bound of the lightest neutral Higgs scalar, for example. In the remaining part of this section we estimate the lightest neutral Higgs mass bound in the present scenario by imposing the condition of the weak scale radiative symmetry breaking. We compare these results with the ones of other models with $\tilde{m}_S^2 > 0$ where the μ-scale appears around the weak scale.

At first we discuss the model with an extra U(1). In this model the tree level scalar potential including the soft supersymmetry breaking terms is the same as the usual $\tilde{m}_S^2 > 0$ case and can be written as[10]

$$\begin{aligned} V_0 = &\frac{1}{8}\left(g_2^2+g_1^2\right)\left(|H_1|^2-|H_2|^2\right)^2 + \left(|\lambda S H_1|^2 + |\lambda S H_2|^2\right) + \\ &+\tilde{m}_{H_1}^2|H_1|^2 + \tilde{m}_{H_2}^2|H_2|^2 + (A_\lambda \lambda S H_1 H_2 + \text{h.c.}) + \\ &+\frac{1}{8}g_E^2\left(Q_1|H_1|^2+Q_2|H_2|^2+Q_S|S|^2\right)^2 + \lambda^2|H_1H_2|^2 + \tilde{m}_S^2|S|^2\,, \end{aligned} \tag{5.1}$$

where Q_1, Q_2 and Q_S are the extra U(1)-charges of H_1, H_2 and S, respectively. The vacuum of these models is parametrized by the vacuum expectation values (VEVs) of Higgs scalar fields such as

$$\langle H_1\rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle H_2\rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \qquad \langle S\rangle = u\,, \tag{5.2}$$

where v_1 and v_2 are assumed to be positive and $v_1^2 + v_2^2 = v^2 (\equiv (174\,\text{GeV})^2)$ should be satisfied. Thus the vacuum in these models is parametrized by $\tan\beta = v_2/v_1$ and u. Based on this scalar potential the upper mass bound of the lightest neutral Higgs scalar is estimated as

$$m_{h^0}^{(0)2} \le m_Z^2\left[\cos^2 2\beta + \frac{2\lambda^2}{g_1^2+g_2^2}\sin^2 2\beta + \frac{g_E^2}{g_1^2+g_2^2}\left(Q_1\cos^2\beta + Q_2\sin^2\beta\right)^2\right]. \tag{5.3}$$

In this derivation we have used the following potential minimization conditions for V_0 :

$$\begin{aligned} \tilde{m}_{H_1}^2 &= -\frac{1}{4}(g_2^2+g_1^2)(v_1^2-v_2^2) - \frac{1}{4}g_E^2 Q_1(Q_1v_1^2+Q_2v_2^2+Q_Su^2) - \lambda^2(u^2+v_2^2) - \\ &\quad -\lambda A_\lambda u\frac{v_2}{v_1}\,, \\ \tilde{m}_{H_2}^2 &= \frac{1}{4}(g_2^2+g_1^2)(v_1^2-v_2^2) - \frac{1}{4}g_E^2 Q_2(Q_1v_1^2+Q_2v_2^2+Q_Su^2) - \lambda^2(u^2+v_1^2) - \lambda A_\lambda u\frac{v_1}{v_2}\,, \\ \tilde{m}_S^2 &= -\frac{1}{4}g_E^2 Q_S(Q_1v_1^2+Q_2v_2^2+Q_Su^2) - \lambda^2(v_1^2+v_2^2) - \lambda A_\lambda\frac{v_1v_2}{u}. \end{aligned} \tag{5.4}$$

[10] Hereafter we will use the same notation for the superfield and its scalar component. The sign convention of A parameters is followed to the one in the previous sections which is reversed from that in refs. [13, 44].

These formulas can be extended so as to include the one-loop effective potential

$$V_1 = \frac{1}{64\pi^2}\mathrm{Str}\ \mathcal{M}^4\ \left(\ln\frac{\mathcal{M}^2}{\Lambda^2} - \frac{3}{2}\right), \tag{5.5}$$

where $\mathcal{M}^2$ is a matrix of the squared mass of the fields contributing to the one-loop correction and Λ is a renormalization point. If we include the effect of V_1, the upper mass bound of the lightest Higgs scalar is modified by adding the following term to the right-hand side of eq. (5.3) :

$$\Delta m_{h^0}^2 = \frac{1}{2}\left(\frac{\partial^2 V_1}{\partial v_1^2} - \frac{1}{v_1}\frac{\partial V_1}{\partial v_1}\right)\cos^2\beta + \frac{1}{2}\frac{\partial^2 V_1}{\partial v_1 \partial v_2}\sin 2\beta + \frac{1}{2}\left(\frac{\partial^2 V_1}{\partial v_2^2} - \frac{1}{v_2}\frac{\partial V_1}{\partial v_2}\right)\sin^2\beta\,. \tag{5.6}$$

This bound is estimated at the minimum of the one-loop effective potential $V_{\rm eff} = V_0 + V_1$.

In order to proceed this estimation we will use the following method which was adopted in ref. [13]. As usual, we evolve the coupling constants and the soft supersymmetry breaking parameters from the suitable initial values at the unification scale M_X towards the low energy region by using the renormalization group equations (RGEs). We can determine $\tan\beta$ using the data on the top quark mass m_t and the top Yukawa coupling constant $h_t(m_t)$.[11] Once $\tan\beta$ is fixed, we can determine the value of u which minimizes the effective potential $V_{\rm eff}$. However, this set of $(\tan\beta, u)$ is not necessarily a true potential minimum. In order to guarantee that these values are indeed obtained at the potential minimum we substitute these values of $(\tan\beta, u)$ into potential minimum conditions (5.4) improved by V_1 and compare them with the corresponding direct low energy results of the RGEs. If these are equal within the suitable range, we recognize such $(\tan\beta, u)$ as the true vacuum and the corresponding parameters set as the phenomenologically allowable one. Only for such parameters set we estimate the upper mass bound of the lightest Higgs scalar m_{h^0} which is defined as $m_{h^0}^2 = m_{h^0}^{(0)2} + \Delta m_{h^0}^2$. In this calculation we change the RGEs from the ones of the supersymmetric model to the ones of the nonsupersymmetric model at $m_{3/2}$. The renormalization point in eq. (5.5) is taken as m_t, for simplicity. Although these are the main points of our analysis, we should mention that we additionally impose further phenomenological conditions such as the mass bounds of superparticles to restrict the parameters, which have also been used in the analysis of [13].

In our analysis we have used first eqs. (3.6) and (3.8) for the initial values of the soft supersymmetry breaking parameters as representative cases of $\tilde{m}_S^2 > 0$ and $\tilde{m}_S^2 < 0$, respectively. However, it seems to be rather difficult to obtain consistent radiative symmetry breaking solutions for the universal soft Higgs masses in the case

[11] To determine the value of $\tan\beta$ an effect of the translation of the running mass to the pole mass has been taken into account [43].

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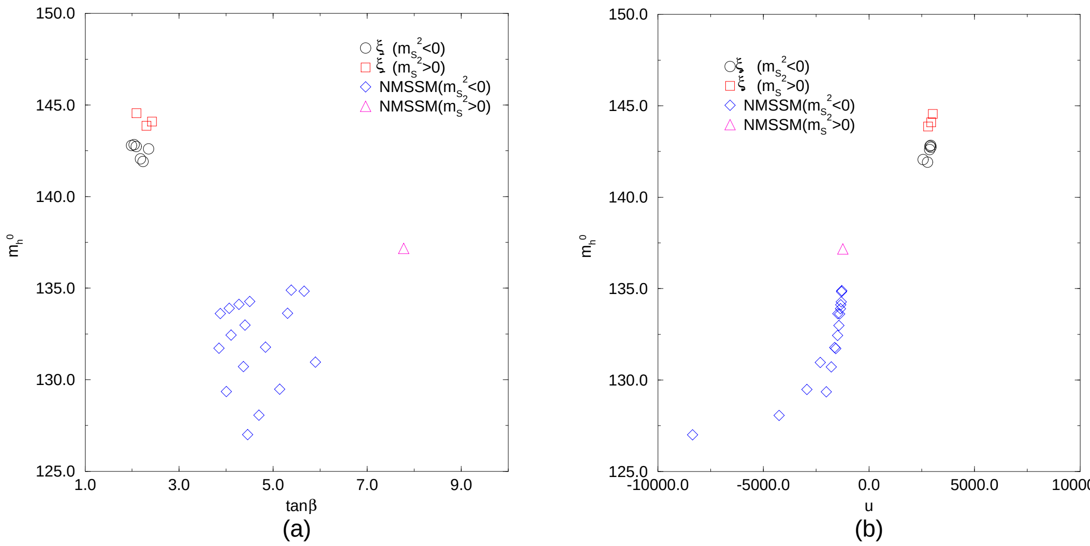

Figure 1: Scatter plots of the upper mass bound m_{h^0} of the lightest neutral Higgs scalar for $\tan\beta$ in figure 1(a) and for u in figure 1(b) for both of the ξ_- model and the NMSSM. They are plotted in both cases of $\tilde{m}_S^2 > 0$ and $\tilde{m}_S^2 < 0$.

of $\tilde{m}_S^2 > 0$. Therefore, we are led to use eq. (3.7) instead of eq. (3.6) in order to represent the $\tilde{m}_S^2 > 0$ case. On the other hand as a typical model with an extra U(1) we take the three generations ξ_- model studied in ref. [13] which is derived from E_6 using the Wilson line breaking mechanism. The SM gauge coupling unification is guaranteed for the field contents of this model since the chiral superfield contents are composed of the MSSM one with $3(\mathbf{5}+\bar{\mathbf{5}})$ of SU(5). The relevant Yukawa couplings to the RGEs analysis are the top Yukawa coupling h_t, λ and k.[12] They are surveyed in the range from 0 to 3 with the interval 0.1, respectively. The results of this analysis for $m_{3/2} = 1\,\text{TeV}$ are summarized in figure 1.

The allowed parameter space seems to be strongly restricted in the case of $\tilde{m}_S^2 > 0$ so that the number of solutions is limited as compared to the one of $\tilde{m}_S^2 < 0$. The sign of $\tilde{m}_S^2$ at the unification scale affects largely the radiative symmetry breaking phenomena. It seems to bring different features for the running of the coupling constants in both cases, although there is no large difference in their initial values at the unification scale. The consistent radiative symmetry breaking tends to occur much more easily in the case of $\tilde{m}_S^2 < 0$. The upper mass bound m_{h^0} takes its value at ∼142 GeV ($\tilde{m}_S^2 < 0$ case) and ∼144 GeV ($\tilde{m}_S^2 > 0$ case). We might expect that the $\tilde{m}_S^2 < 0$ case generally gives smaller values for the m_{h^0} as compared to the $\tilde{m}_S^2 > 0$ one. We also carried out the calculation for other values of $m_{3/2}$. The qualitative feature is very similar, although the numerical value of m_{h^0} tends to be shifted upwards for the larger value of $m_{3/2}$. It is also useful to note that in the

[12]We may consider the model with no extra colored fields $(g, \bar{g})$ in the $\tilde{m}_S^2 < 0$ case as an interesting alternative. We do not consider such a case here since further consideration on the matter content for the anomaly cancellation is required.

model with an extra U(1) values of $\tan\beta$ smaller than 2 are allowed even for the present Higgs mass bound since there is the extra contribution depending on λ as shown in eq. (5.3).[13]

Next we proceed to the analysis of the NMSSM. The NMSSM is another candidate to solve the μ-problem in the MSSM. As mentioned in the introduction, the discrete symmetry should be imposed in the NMSSM to escape the phenomenological problems such as the bare μ-term and the tadpole.

Although, in general, the spontaneous breaking of the discrete symmetry generates the cosmologically dangerous domain wall problem, we leave it here. We estimate the upper mass bound of the Higgs scalar also in the NMSSM paying attention to the difference between the $\tilde{m}_S^2 < 0$ and $\tilde{m}_S^2 > 0$ cases. The NMSSM is composed of the MSSM content and a singlet chiral superfield. Here we do not consider the introduction of the extra matter fields such as $5+\bar{5}$ of SU(5).[14] In the superpotential we should replace $kSg\bar{g}$ in the ξ_- model to $\frac{k}{3}S^3$. As the supersymmetry breaking parameters in this study, we use those given in eq. (3.7) with $A_k/k = (\sqrt{33}+3\sqrt{2}-2\sqrt{6})m_{3/2}/6$ and the ones in eq. (3.8) with $A_k/k = (5-2\sqrt{2})m_{3/2}/\sqrt{3}$ for the cases of $\tilde{m}_S^2 > 0$ and $\tilde{m}_S^2 < 0$, respectively. The Yukawa couplings h_t, $|k|$ and λ are serveyed in the range from 0 to 3 with an interval 0.1, while $m_{3/2}$ is fixed to be 1 TeV.

The results of this analysis are also presented in figure 1. The allowed parameter space again seems to be strongly restricted in the $\tilde{m}_S^2 > 0$ case as compared to the $\tilde{m}_S^2 < 0$ case. Since there is no direct contribution to the RGE of $\tilde{m}_S^2$ from the Yukawa coupling which is affected by the SU(3) gauge coupling in the NMSSM, it is rather difficult to make $\tilde{m}_S^2$ negative radiatively and this tendency becomes much clearer as compared to the model with an extra U(1). The upper mass bound m_{h^0} takes its value in the region of 127 GeV$\sim$135 GeV ($\tilde{m}_S^2 < 0$ case) and $\sim$137 GeV ($\tilde{m}_S^2 > 0$ case). Also in the NMSSM the value of m_{h^0} is smaller in the $\tilde{m}_S^2 < 0$ case. In this case the smaller value of m_{h^0} tends to be realized at the large $|u|$ region where there is no solutions in the $\tilde{m}_S^2 > 0$ case. Since only the effect of k and λ cannot make the value of $\tilde{m}_S^2$ negative largely, the negative initial value of $\tilde{m}_S^2$ seems to be necessary to derive the larger $|u|$ value in the NMSSM. The large $|u|$ value tends to be combined with the smaller λ since λu is related to the μ-scale. This feature suggests that the $\tilde{m}_S^2 < 0$ model can have the possibility to realize a smaller upper mass bound of m_{h^0}.

Up to now we focussed our attention on the soft supersymmetry breaking parameters which could be derived in the superstring framework. As we have discussed already, the soft supersymmetry breaking scalar masses appear also in the field theoretical framework and the corresponding sum rule (4.22) can be obtained there too assuming the reduction of couplings scenario. In the latter framework we shall study the effect of $\tilde{m}_S^2$ on the Higgs mass bound under the constraint coming from the

[13] The results shown in figure 1 are obtained for rather restricted soft parameters and then this aspect can be not seen explicitly. This feature of allowed $\tan\beta$ will be expected in the NMSSM [13, 44].

[14] The inclusion of these is expected to raise the Higgs upper mass bound as discussed in [44].

sum rule. We should note that there are some noticeable points which are characteristic in this study. The Gauge-Yukawa and Finite SU(5) models [23, 24], which have been so far successful in predicting, among others, the top quark mass require large values of $\tan\beta$ since the ratio of the top and bottom Yukawa couplings h_t and h_b (as well their ratio to the GUT gauge coupling, $g_{\rm GUT}$) are of order one at the unification scale. This situation is completely different from the above superstring case, where the sum rule is required only for the Yukawa couplings which satisfy a condition such as (2.22). However the field theory framework, according to the partial reduction of couplings proposal [41], is flexible on the choice of the couplings that should be reduced. Therefore one can always reduce the number of the sum rules that have to be satisfied. Therefore, as a first try, assuming that a successful GUT could be constructed with reduction of couplings which keeps the above characteristics of the known Gauge-Yukawa GUTs and moreover incorporates a term such as $\lambda S H_1 H_2$ with a coupling which is reduced in favor of the corresponding $g_{\rm GUT}$, we shall study the consequences of the sum rule applied in this term.[15] Then the relation $\tilde{m}^2_{H_1} + \tilde{m}^2_{H_2} + \tilde{m}^2_S = M^2$ should be satisfied and clearly the value of $\tilde{m}^2_S$ reflects to those of $\tilde{m}^2_{H_1}$ and $\tilde{m}^2_{H_2}$ which are already constrained from the large $\tan\beta$. We shall see in the following that, as a result of imposing these constraints, it becomes completely non-trivial to achieve radiative electroweak symmetry breaking.

Under the assumptions described above we take the NMSSM within the reduction of couplings scenario as the target of our numerical study. Then in the present RGE study we need to include the bottom and tau Yukawa couplings h_b, h_τ since they are comparable to h_t and we assume the validity of the sum rule corresponding to all Yukawa couplings of the third generation, assuming reduction of the corresponding couplings.[16] We can further impose the condition $h_b = h_\tau$ at the unification scale as resulting naturally in most GUTs. Then we would like to determine which sum rules, as those suggested by eq. (4.22), can be imposed at the unification point and simultaneously are compatible with the low energy phenomenolological conditions. As a numerical strategy to find solutions to this requirement we use the sum rule condition in the form[17] of $A_{IJK} = -M h_{IJK}$ and $\tilde{m}^2_I + \tilde{m}^2_J + \tilde{m}^2_K = \mathcal{C} M^2$, in which $\mathcal{C}$ is treated as the free parameter satisfying $0 \le \mathcal{C} \le 3$.[18] Moreover, we treat soft

[15] Not hat a singlet cannot have non-vanishing couplings in Finite N = 1 GUTs, resulting from the finiteness condition that requires vanishing of the corresponding anomalous dimension of the chiral singlet field. This is not the case in other Gauge-Yukawa GUTs.

[16] Note that the coupling of a term such as $\frac{k}{3} S^3$ cannot be reduced in favor of the GUT gauge coupling, since S is gauge singlet.

[17] Note that we use the notation of section 2 here.

[18] We assume $\mathcal{C}_t = \mathcal{C}_b$ which is the case for the grand unified model with the reduction of couplings. For the τ Yukawa coupling, however, $\mathcal{C}_\tau$ is fixed as 3 and also $\tilde{m}^2_{\bar{E}} = M^2$ since the small $\mathcal{C}_\tau$ tends to violate the superparticle mass bound, in particular, the chargino mass bound and the electric charge conservation. This obviously consists an additional difficulty in finding the prescribed solutions.

	$\tan\beta$	u (TeV)	m_{h^0} (GeV)	$\mathcal{C}_\lambda$	$\mathcal{C}_t$
$\tilde{m}_S^2 > 0$	~ 28.2	$1.41 \sim 1.80$	$135.6 \sim 139.8$	$0.82 \sim 2.13$	$1.0 \sim 3.0$
$\tilde{m}_S^2 < 0$	~ 40.1	$1.74 \sim 2.26$	$137.6 \sim 142.6$	$0.36 \sim 1.48$	$1.5 \sim 3.0$

Table 1: The numerical results in the field theory case.

scalar masses as the independent free parameters and vary $|\tilde{m}_I|$ from $0.1M$ to M, although the gaugino masses are treated as the universal value M. We impose the constraint of consistent realization of the bottom and tau mass at the low energy scale in addition to the previously explained procedure. We take $m_{3/2} = 1\,\mathrm{TeV}$ as the scale where we change the set of RGEs and the gaugino mass M is varied from $0.7m_{3/2}$ to $m_{3/2}$. Yukawa couplings are serveyed in the similar region to the previous superstring case.

The solutions of this numerical calculation are presented in table 1. We could obtain these solutions only for the very restricted values for the Yukawa couplings

$$\begin{aligned} &h_t \simeq 1.06, \qquad h_b \simeq 0.32, \qquad k \simeq 0.3, \qquad \lambda \simeq 2.4, \qquad (\tilde{m}_S^2 > 0), \\ &h_t \simeq 1.08, \qquad h_b \simeq 0.54, \qquad k \simeq 0.4, \qquad \lambda \simeq 2.1, \qquad (\tilde{m}_S^2 < 0) \end{aligned} \tag{5.7}$$

at the unification scale. The large $\tan\beta$ solution requires fine tuned Yukawa couplings for a consistent electroweak radiative symmetry breaking. These are easily realized for the non-universal soft Higgs scalar masses $\tilde{m}_{H_1}^2 \neq \tilde{m}_{H_2}^2$. In the $\tilde{m}_S^2 < 0$ case we have solutions only for $\tilde{m}_{H_1}^2 \geq \tilde{m}_{H_2}^2$ although in the $\tilde{m}_S^2 > 0$ case we can obtain solutions even for $\tilde{m}_{H_1}^2 < \tilde{m}_{H_2}^2$. The value of $\tan\beta$ is rather different for different sign of $\tilde{m}_S^2$. This situation is very different from the previous examples in which $\tan\beta$ takes similar value in both cases. This may be reflected to the upper mass bound of the neutral Higgs scalar which is a little bit smaller for the $m_S^2 > 0$ case as compared to the $m_S^2 < 0$ case. Although we do not strictly impose the sum rule for the soft scalar masses at the unification scale, we can see whether it is satisfied in these solutions through the values of $\mathcal{C}$ in table 1. The value of $\mathcal{C}$ shown in table 1 is found to reflect the sign of $\tilde{m}_S^2$. Since the negative $\tilde{m}_S^2$ tends to make $\tilde{m}_{H_{1,2}}^2$ larger, we can expect that the solutions tend to have smaller $\mathcal{C}_\lambda$ and larger $\mathcal{C}_t$ as compared to the positive $\tilde{m}_S^2$. In fact, many solutions of the negative $\tilde{m}_S^2$ case satisfy $\mathcal{C}_\lambda < 1$ and $\mathcal{C}_t > 1$. Therefore our study shows that we cannot apply consistently our scenario if we insist in reducing the Yukawa couplings of the third generation in favor of g_{GUT}, which would imply the validity of the corresponding sum rules. Therefore we are led to abandon the freedom offered by the field theory in reducing more couplings and we keep only as an interesting viable possibility the reduction of the coupling λ of the term $\lambda S H_1 H_2$, which in turn implies a sum rule as in the string case discussed earlier. It is amusing that in the positive $\tilde{m}_S^2$ case many solutions satisfy the sum rules and then $\mathcal{C}_\lambda \sim \mathcal{C}_t \sim 1$ in rather good accuracy. If we consider the couplings reduction in the type of model as the NMSSM, the positive $\tilde{m}_S^2$ will be promising and $\tan\beta \sim 30$ may be expected.

6. Summary and conclusions

We have considered the possible solution for the μ-problem based on the non-universal soft supersymmetry breaking scalar masses. Usually models with negative soft squared mass are excluded since they are led to incorrect SM vacuum. However, if we consider the non-universality of the soft supersymmetry breaking parameters in the extended models of the MSSM seriously, we can obtain viable models for some sets of their initial conditions. Here we have proposed such a kind of application of the non-universal soft supersymmetry breaking to resolve the μ-problem.

In this paper we have concretely showed the possibilities that the negative soft scalar squared mass were constructed consistently in the superstring framework. We have also discussed that in the field theory with the reduction of couplings the non-universal soft scalar masses could be realized satisfying the suitable sum rules. In both cases after the extension of the MSSM it seems not to be difficult to realize the negative squared soft scalar mass in the extended chiral superfield sector keeping its positivity in the sector of the MSSM at the unification scale. The question is whether the radiative electroweak symmetry breaking can occur starting from such a kind of initial conditions. We have studied to which extent this scenario could be consistent with the electroweak radiative symmetry breaking. Our numerical study shows that indeed the realization of such a scenario is possible, although the solutions seem to be shifted in the parameters space as compared to the usual case with the positive squared soft scalar masses. As a result of this shift in the parameters space, we also suggested that in the superstring case the upper mass bound of the lighest neutral Higgs scalar could be smaller in this type of models than the ones with the positive squared soft scalar masses. On the other hand, in the field theory case if we insist in reducing in favor of the gauge coupling also the Yukawa coupligs of the third generation, in addition to the coupling of the singlet to the Higgs fields, the sum rules for the soft supersymmetry breaking parameters can be easily satisfied only in the case of positive soft scalar squared masses. Certainly a negative soft scalar squared mass corresponding to the singlet superfield can also be obtained by relaxing the requirement to reduce the Yukawa couplings in favor of the gauge couplings.

These results seem to be interesting enough for the extended MSSM with an SM singlet chiral superfield. If we take into account the new possibility discussed here, the extended MSSM with the SM singlet chiral superfield may reveal the new aspect of its phenomenology and be a similarly promising candidate to the MSSM, for example, in the Higgs search. Our present scenario for the origin of the μ-scale seems to be a simple example of the phenomena that the non-universal soft supersymmetry breaking can introduce in the theory and might exist further fruitful applications in the model building beyond the MSSM.

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